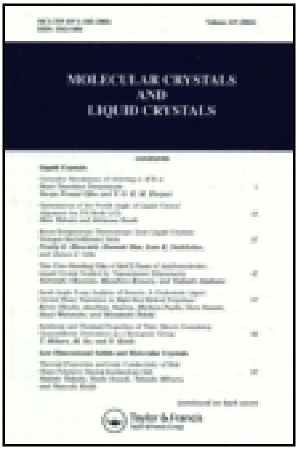

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The peculiarities of light propagation in uniaxial chiral media with large pitch are studied. In this case there are the forbidden zones for extraordinary beams. The existence of the forbidden zones leads to the effective reflection on zones boundaries and to damping of the wave inside the forbidden zone. We analyze the vicinities of the turning points and the transition of an extraordinary wave through the forbidden zone. It is shown that the optical effects for beams propagating near the turning points are equivalent to the tunnelling and reflection above the barrier for the parabolic potential barrier in quantum mechanics.

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Keywords: forbidden zones; helical liquid crystal; turning points; wave-guide channel

1. INTRODUCTION

The designing of various mapping systems and devices of transmitting an information stimulate the particular interest to the study of the optical properties of liquid crystals [1–3]. As a result there emerged new problems which were almost neglected in physical optics. One of such problems is the study of light propagation in anisotropic media with smoothly varying properties. Among such media are nematic liquid crystals in inhomogeneous electric and magnetic fields, ferroelectric smectic C^*, nematic twist-cells, cholesteric liquid crystals, etc.

For a long time this problem was investigated for the systems with the pitch less or of the order of the wave length [4]. Within this purpose effective numerical methods were suggested for calculation

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of the wave transmission and reflection in chiral liquid crystals [5,6] and nematic twist-cells [7]. The exact solution was obtained for the wave propagation along the pitch [8,9]. For the oblique incidence the effective approximate methods based on the Floquet theorem and methods of diffraction theory were developed [10]. Up to now these approaches are widely used for the study of wave propagation in the chiral systems with small pitch [11,12]. Their efficiency is confirmed in numerous experimental studies.

In this contribution we present a systematic theoretical study of wave propagation in chiral media in case when the pitch significantly exceeds the wavelength. The problem of electromagnetic wave propagation in locally isotropic systems with smoothly varying optical properties has been solved in [13,14]. From the point of view of physical optics the chiral media are the uniaxial systems with periodically varying direction of the optical axis. A wave in such medium can be considered as an ordinary and an extraordinary wave varying in accordance to the local optical parameters. In the geometrical optics approximation the extraordinary beam propagates in this region as in a layered medium with periodically varying refractive index. For the large angle of incidence the return back of the extraordinary beam in these layered system is observed [15]. It relates to forming of the forbidden zones periodically interlacing with the regions of waves propagation.

The emphasis is made on successive description of the wave transmission through the forbidden zones and calculation of transmission and reflection coefficients in relation to the angle of incidence. The problem is solved by application of the vector analog of the WKB (Wentzel-Kramers-Brillouin) method and of the standard equation near the turning points.

2. ASYMPTOTIC SOLUTION OF THE WAVE EQUATION FOR THE CHIRAL MEDIA

In chiral liquid crystal (LC) the preliminary direction of the long axes of molecules is determined by the unit vector director $\mathbf{n}^0$. We consider the plane parallel LC twist-cell. In this case vector director rotates along the axis normal to the cell forming a spiral. It is suitable to introduce the coordinate frame with z axis directed along the spiral axis so that vector director rotates in (x,y) plane with variation of z

$$\mathbf{n}^0(z) = (\cos(q_0 z + \phi_0), \sin(q_0 z + \phi_0), 0), \tag{2.1}$$

where the angle ϕ_0 determines the direction of the director in the plane $z = 0$, $q_0 = 2\pi/p_0$, p_0 is the pitch. As far as the directions $\mathbf{n}^0(z)$

and $-\mathbf{n}^0(z)$ are equivalent $p_0/2$ determines the period of the optical properties in the twist-cell.

Optical properties of the LC are determined by the local uniaxial permittivity tensor [8]

$$\varepsilon_{lk}(z) = \varepsilon_\perp \delta_{lk} + \varepsilon_a n_l^0(z) n_k^0(z), \quad l,k = 1,2,3. \tag{2.2}$$

Here $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$, $\varepsilon_\parallel$, $\varepsilon_\perp$ are permittivities along and transverse to $\mathbf{n}^0$. The system with $\varepsilon_a > 0$ and $\varepsilon_a < 0$ are considered in the similar way. For definiteness we consider LC with $\varepsilon_a > 0$. The Maxwell equation set has the form

$$\operatorname{curl} \mathbf{E}(\mathbf{r}) = ik_0 \hat{\mu} \mathbf{H}(\mathbf{r}), \quad \operatorname{curl} \mathbf{H}(\mathbf{r}) = -ik_0 \hat{\varepsilon}(z) \mathbf{E}(\mathbf{r}), \tag{2.3}$$

where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields, $k_0 = \omega/c$, ω is the circular frequency, c is the light velocity in vacuum, $\hat{\mu}$ is the tensor of the magnetic permittivity. We suppose the system to be considered is nonmagnetic so that $\mu_{\alpha\beta} = \delta_{\alpha\beta}$. We are interested in the field in the chiral medium with pitch significantly much larger than the wave length. We will solve the Maxwell equation by the WKB method restricting ourself by the zeroth and the first approximation. Besides we consider the ranges where the mode transformations take place. In these ranges the WKB approximation is not valid and the solution of the standard equation should be used.

As far as the permittivity tensor $\hat{\varepsilon}(z)$ depends on z it is convenient to complete the Fourier transformation over transverse variables,

$$F(\mathbf{k}_\perp, z) = \int d\mathbf{r}_\perp F(\mathbf{r}) e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}. \tag{2.4}$$

For convenience we choose $\mathbf{k}_\perp$ vector directed along x axis, i.e., $\mathbf{k}_\perp = (k_\perp, 0, 0)$. Introducing the new variable $\xi = q_0 z + \phi_0$ and excluding H_z and E_z components we get

$$\partial_\xi \begin{pmatrix} E_x \\ E_y \\ H_x \\ -H_y \end{pmatrix} = -i\Omega \begin{pmatrix} 0 & 0 & 0 & \alpha \\ 0 & 0 & 1 & 0 \\ \varepsilon_{xy}(\xi) & \varepsilon_{yy}(\xi) + \varepsilon_\perp(\alpha - 1) & 0 & 0 \\ \varepsilon_{xx}(\xi) & \varepsilon_{xy}(\xi) & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ H_x \\ -H_y \end{pmatrix}, \tag{2.5}$$

where $\partial_\xi = \partial/\partial\xi$, $\alpha = 1 - k_\perp^2/(k_0^2 \varepsilon_\perp)$, $\Omega = k_0/q_0 = p_0/\lambda$, $\lambda = 2\pi/k_0$ is the wave length.

The equation set (2.5) can be written formally as the equation for four-component vector-column $\mathbf{\Phi}(\xi)$

$$\partial_\xi \mathbf{\Phi}(\xi) = i\Omega \widehat{A}(\xi) \mathbf{\Phi}(\xi). \tag{2.6}$$

The equation (2.6) can be solved by the WKB method [13,14,16]. If we determine the component E_z from equation $\hat{\varepsilon}\mathbf{E} = 0$ then the field can be written in the form

$$\mathbf{E}_{\pm}^{(j)}(\mathbf{r}) = E_0^{(j)} A^{(j)}(\mathbf{k}_\perp; z, z_0)\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)\exp\left(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp \pm i\int_{z_0}^{z} k_z^{(j)}(\mathbf{k}_\perp, z')dz'\right), \tag{2.7}$$

where $j = o, e$, and where o and e mean ordinary and extraordinary wave respectively, the signs $\pm$ correspond to the direction of the wave propagation. There are four eigenwaves for given $\mathbf{k}_\perp$ in cholesteric liquid crystal with large pitch. Two of them propagate in the direction of the positive z and two waves propagate in the direction of the negative z. The constants $E_0^{(j)}$ determine the initial amplitude of the field in the plane z_0. Here $k_z^{(j)}(\mathbf{k}_\perp, z')$ is the longitudinal component of the wave vector in the plane $z = z'$, $\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)$ are the unit vectors, and $A^{(j)}(\mathbf{k}_\perp; z, z_0)$ are the amplitude factors. All these functions vary smoothly in the scale of the order of the wavelength. The waves (2.7) locally are plane waves with the wave vectors $\mathbf{k}^{(j)}(\mathbf{k}_\perp, z) = (\mathbf{k}_\perp, \pm k_z^{(j)}(\mathbf{k}_\perp, z))$ and polarization vectors $\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)$. In the first order over the large parameter Ω we have

$$k_z^{(o)}(\mathbf{k}_\perp, z) \equiv k_z^{(o)}(k_\perp) = \sqrt{\varepsilon_\perp k_0^2 - k_\perp^2} = k_0\sqrt{\varepsilon_\perp \alpha}, \tag{2.8}$$

$$k_z^{(e)}(\mathbf{k}_\perp, z) = \sqrt{\varepsilon_\parallel k_0^2 - k_\perp^2 - \frac{\varepsilon_a}{\varepsilon_\perp}(\mathbf{k}_\perp \cdot \mathbf{n}^0(z))^2} = k_0\sqrt{\alpha\varepsilon_\parallel + (1-\alpha)\varepsilon_a \sin^2\xi}, \tag{2.9}$$

$$A^{(o)}(\mathbf{k}_\perp; z, z_0) = 1,$$
$$A^{(e)}(\mathbf{k}_\perp; z, z_0) = \sqrt{\frac{\varepsilon_\perp^2 k_0^2 + \varepsilon_a(\mathbf{k}_\perp \cdot \mathbf{n}^0(z))^2}{\varepsilon_\perp^2 k_0^2 + \varepsilon_a(\mathbf{k}_\perp \cdot \mathbf{n}^0(z_0))^2}}\sqrt{\frac{k_z^{(e)}(\mathbf{k}_\perp, z_0)}{k_z^{(e)}(\mathbf{k}_\perp, z)}}, \tag{2.10}$$

and the polarization vectors $\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)$ in the introduced coordinate frame are

$$\mathbf{e}^{(o)\pm}(\mathbf{k}_\perp, z) = \frac{1}{\sqrt{k_0^2\varepsilon_\perp - k_\perp^2\cos^2\xi}}(\pm k_z^{(o)}\sin\xi,\ \mp k_z^{(o)}\cos\xi,\ -k_\perp\sin\xi),$$
$$\mathbf{e}^{(e)\pm}(\mathbf{k}_\perp, z) = \frac{\sqrt{\varepsilon_\perp}}{\sqrt{(k_0^2\varepsilon_\perp - k_\perp^2\cos^2\xi)(k_0^2\varepsilon_\perp^2 + \varepsilon_a k_\perp^2\cos^2\xi)}}$$
$$\times\ ((k_0^2\varepsilon_\perp - k_\perp^2)\cos\xi,\ k_0^2\varepsilon_\perp\sin\xi,\ \mp k_\perp k_z^{(e)}\cos\xi). \tag{2.11}$$

As far as the vectors $\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)$ in (2.11) are real in the first order over Ω the waves (2.7) are plane polarized waves. In the next order over Ω there appears an imaginary addition in $\mathbf{e}^{(j)\pm}(\mathbf{k}_\perp, z)$ and the waves (2.7) obtain a weak ellipticity.

In each point the polarization of the ordinary wave $\mathbf{e}^{(o)\pm}$ is perpendicular to the plane containing the vectors $\mathbf{k}^{(o)}$ and $\mathbf{n}^0$. The polarization vector of the extraordinary wave $\mathbf{e}^{(e)\pm}$ is in the plane formed by the vectors $\mathbf{k}^{(e)}$, $\mathbf{n}^0$ and its direction is determined by the condition $\hat{\varepsilon}\mathbf{e}^{(e)\pm} \perp \mathbf{k}^{(e)}$.

It is well known that the solution of the equation set (2.3) for homogeneous medium with uniaxial permittivity tensor presents four electromagnetic waves [17], two ordinary and two extraordinary ones

$$\mathbf{E}^{(j)\pm} = E_0 \mathbf{e}^{(j)\pm} \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp \pm i k_z^{(j)} z). \tag{2.12}$$

The refraction indices of these waves are given by the relations

$$n^{(o)} = \sqrt{\varepsilon_\perp}, \quad n^{(e)} = \sqrt{\frac{\varepsilon_\perp \varepsilon_\parallel}{\varepsilon_\parallel \cos^2\theta + \varepsilon_\perp \sin^2\theta}}, \tag{2.13}$$

where $n^{(o)}$, $n^{(e)}$ are refraction indices of the ordinary and extraordinary wave respectively, θ is the angle between the wave vector $\mathbf{k}$ and the direction of the optical axis.

It should be noted that the solution (2.7) in each point can be considered as a plane wave in uniaxial homogeneous medium with varying optical parameters. For the ordinary wave the index of refraction is a constant whereas for extraordinary wave the index of refraction depends on z. The waves (2.7) are propagating if $k_z^{(j)}$ is real only. The propagation of the waves in these media depends on the transverse component of the wave vector $k_\perp$.

We consider the case $\varepsilon_a > 0$. In this case $k_\perp$ varies within the limits $0 \le k_\perp^2 \le k_0^2\varepsilon_\parallel$ (see Fig. 1). Let us analyze the behavior of $k_z^{(o)}$ and $k_z^{(e)}$ as functions of $k_\perp$. For this purpose it is convenient to use α parameter varying in the interval $-\varepsilon_a/\varepsilon_\perp \le \alpha \le 1$. As it follows from Eqs. (2.8) and (2.9) for $0 < \alpha \le 1$ ($0 \le k_\perp^2 < k_0^2\varepsilon_\perp$) the radicand in $k_z^{(o)}$ and $k_z^{(e)}$ is positive for any ξ and therefore both waves propagate everywhere in the twist cell (Fig. 2(a)). For $-\varepsilon_a/\varepsilon_\perp \le \alpha < 0$ ($k_0^2\varepsilon_\perp \le k_\perp^2 < k_0^2\varepsilon_\parallel$) the radicand in (2.8) is negative for any ξ whereas the radicand in (2.9) is negative in certain interval of ξ due to rotation of the permittivity ellipsoid. In this case $k_z^{(o)}$ is imaginary and ordinary wave decays exponentially in the whole of the twist cell. The exponential decay for extraordinary wave takes place in the restricted regions of imaginary $k_z^{(e)}$ (Fig. 2(b)). Inside these regions the regime of propagation of wave

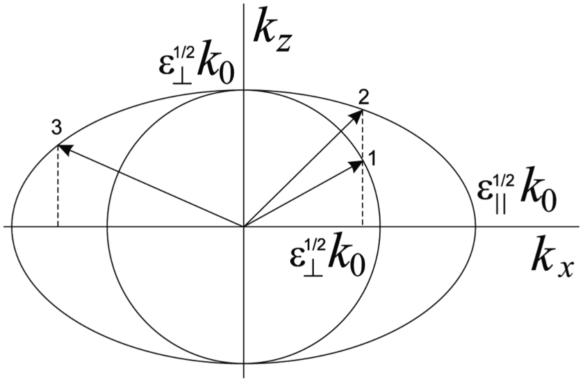

FIGURE 1 The cross section of the wave vector surface for $\varepsilon_a > 0$, the cross section is made for the $\xi = 0$. While ξ varying the surface of the ordinary beam (sphere) does not change whereas the surface of the extraordinary beam (ellipsoid) changes its form since the length of the semi-axis varies. In particular, for changing of ξ by $\pi/2$ the ellipsoid transforms into the sphere. The number of waves propagating in the medium depends on the value of $k_\perp$: vectors 1 and 2 corresponds to the case of two propagating waves; vector 3 – to the case of one propagating wave.

in ξ direction is impossible and therefore these regions present the forbidden zones. As it follows from (2.9) the position of the forbidden zone is determined by the inequality

$$\sin^2 \xi \leq -\frac{\alpha}{1-\alpha}\frac{\varepsilon_\parallel}{\varepsilon_a}. \qquad (2.14)$$

In the case of equality in Eq. (2.14) this equation determines the points with $k_z^{(e)} = 0$ where the wave vector is directed along the x axis. The return back of the extraordinary wave takes place in these points.

The presented arguments are based on the expressions for the field obtained by the WKB approximation. However if denominator of Eq. (2.10) tends to zero then the amplitudes of the obtained solutions tend to infinity and the WKB method is actually not valid in the vicinity of these points. As it follows from analysis of the WKB method applicability, the values of the longitudinal components of the wave vectors $\pm k_z^{(o)}$ and $\pm k_z^{(e)}$ should not approach each other. In particular $k_z^{(e)}$ can not be close to zero. The points where the longitudinal components of the wave vectors coincide are designed as degenerated or turning points. In the vicinity of the turning points the mutual transformation of modes, or exchange of the energy, is possible.

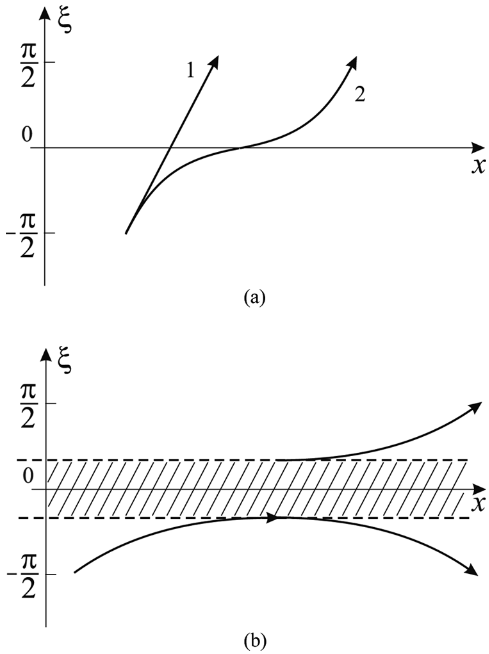

FIGURE 2 Projection of the extraordinary beam trajectory in the plane *xz* and the trajectory of the ordinary beam in the chiral medium for various angles of incidence: (a) the transmitted ordinary, 1, and the extraordinary, 2, beams for $\varepsilon_a > 0$, (b) the forbidden zone for the extraordinary beam (shaded area).

As it follows from Eq. (2.9) the degeneration points $k_z^{(e)} = -k_z^{(e)} = 0$ are determined from the relation

$$\sin^2 \xi_* = -\frac{\alpha}{1-\alpha}\frac{\varepsilon_\parallel}{\varepsilon_a}. \tag{2.15}$$

In these points $k_z^{(e)}(\xi_*) = 0$ and the wave vector is directed along the x axis. As far as the components $k_z^{(e)}$ and $-k_z^{(e)}$ determine the propagation of the extraordinary waves in the vicinity of the point ξ_* the interaction of the incident and reflected extraordinary waves takes place. In what follows this type of interaction will be denoted by $e^+ - e^-$. In the case of $\varepsilon_a > 0$ the degeneration of $k_z^{(o)} = k_z^{(e)}$ type is possible. It takes place in the points satisfying to the condition

$$\sin^2 \xi_{**} = -\alpha/(1-\alpha). \tag{2.16}$$

The waves of (o) and (e) type interact in these points. This type of interaction we denote as $e - o$.

In the vicinity of the points ξ_*, ξ_{**} the solution (2.7) is not correct. The WKB approximation is not valid in the region where the correction of the next order over Ω and the solution (2.7) are comparable. Hence the WKB approximation is violated in the regions determined by inequalities

$$|\xi - \xi_*| \lesssim \Omega^{-2/3} \tag{2.17}$$

and

$$|\xi - \xi_{**}| \lesssim \Omega^{-1/2}. \tag{2.18}$$

The existence of the forbidden zones leads to singularities in the propagation of the waves in the chiral media. As it is seen from Figure 2(b) the turn back of the beam is similar to the total reflection in the point where the longitudinal component of the wave vector reduces to zero. This effect was revealed experimentally and analyzed theoretically in [15]. Moreover the transmission of the waves through the forbidden zone of finite width is possible. If the width of the forbidden zone

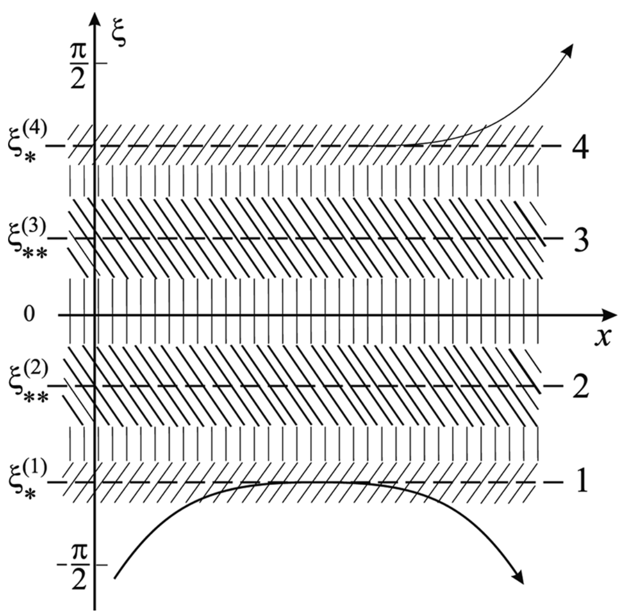

FIGURE 3 The structure of the wide forbidden zone for $\varepsilon_a > 0$. In the regions 1 and 4 two extraordinary beams interact, in the regions 2 and 3 interaction between ordinary and extraordinary beams takes place. The vertical strokes show the regions where the WKB approximation is applied.

is not large then the transmitted wave can be observed experimentally. This effect is similar to the tunnelling effect for electrons in a crystal.

Besides that an emergence of the reflected beam for the transmitted extraordinary beam is possible near the point of inflection (Fig. 2(a)) where the component $k_z^{(e)}$ is small. This effect corresponds to reflection above the barrier. The consistent description of transmission and reflection of waves in the vicinity of forbidden zones requires the analysis of each turning point. In the general case for $\varepsilon_a > 0$ there are four turning points inside the forbidden zone (Fig. 3).

3. THE WIDE FORBIDDEN ZONES

In the case of the wide forbidden zones it is possible to describe the wave behavior in the vicinity of all singular points. Hereinafter we consider one period of LC bounded by the planes $\xi = -\pi/2$ and $\xi = \pi/2$. Each Eqs. (2.15) and (2.16) has two solutions. The modes transformation takes place in the vicinity of the four degeneration points (turning points) shown in Figure 3. We consider that the forbidden zones are so wide that between the turning points 1–4 in Figure 3 there are the regions where the WKB approximation is valid.

The complexity of the analysis of the wave propagation in the vicinity of the turning points is that it is impossible to consider the light field as the set of the ordinary and extraordinary waves with smoothly varying parameters. Near the turning points we must solve the standard equation for the field. In order to get the solution within all ranges including the turning points it is necessary to match WKB solutions and solutions of the standard equation.

In what follows it is convenient to consider equation set for the electric field components instead of Eq. (2.5). The wave equation for vector $\mathbf{E}$ has the form

$$(\text{curl curl} - k_0^2\hat{\varepsilon}(z))\mathbf{E}(\mathbf{r}) = 0. \tag{3.1}$$

For $(\mathbf{k}_\perp, z)$ presentation we have

$$\begin{cases} \partial_\xi^2 E_x - i\Omega\dfrac{k_\perp}{k_0}\partial_\xi E_z + \Omega^2\varepsilon_{xx}E_x + \Omega^2\varepsilon_{xy}E_y = 0 \\ \partial_\xi^2 E_y + \Omega^2\varepsilon_{xy}E_x + \Omega^2\left(\varepsilon_{yy} - \dfrac{k_\perp^2}{k_0^2}\right)E_y = 0 \\ -i\Omega\dfrac{k_\perp}{k_0}\partial_\xi E_x + \Omega^2\varepsilon_\perp\alpha E_z = 0. \end{cases} \tag{3.2}$$

First of all we consider the field in the vicinity of the turning point $\xi_*^{(1)}$, (see Fig. 3), $\sin\xi_*^{(1)} = -\sqrt{-\alpha\varepsilon_\parallel/[(1-\alpha)\varepsilon_a]}$. Excluding the component E_z we get Eq. (3.2) in the form

$$\begin{cases} \partial_\xi^2 E_x + \alpha\Omega^2\varepsilon_{xx}E_x + \alpha\Omega^2\varepsilon_{xy}E_y = 0 \\ \partial_\xi^2 E_y + \Omega^2\left(\varepsilon_{yy} - \dfrac{k_\perp^2}{k_0^2}\right)E_y + \Omega^2\varepsilon_{xy}E_x = 0. \end{cases} \tag{3.3}$$

Near the turning point $\xi_*^{(1)}$ solution corresponding to extraordinary wave is the linear combination of Hankel functions. Asymptotics of the solution in the vicinity of the turning point for $|\xi - \xi_*^{(1)}| \gg \Omega^{-2/3}$ has the form

$$\begin{aligned} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_< &= \sqrt{\frac{3}{\pi}} \frac{\upsilon^{-1/12}}{(\xi_*^{(1)} - \xi)^{1/4}\Omega^{1/6}(\alpha + (1-\alpha)\sin^2\xi_*^{(1)})^{1/2}} \begin{pmatrix} \alpha\cos\xi_*^{(1)} \\ \sin\xi_*^{(1)} \end{pmatrix} \\ &\times \left[A_<^{(1)} \exp\left(\frac{2i}{3}\upsilon^{1/2}\Omega(\xi_*^{(1)} - \xi)^{3/2} - \frac{5i\pi}{12}\right)\right. \\ &\left. + B_<^{(1)} \exp\left(-\frac{2i}{3}\upsilon^{1/2}\Omega(\xi_*^{(1)} - \xi)^{3/2} - \frac{5i\pi}{12}\right)\right], \end{aligned} \tag{3.4}$$

$$\begin{aligned} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_> &= \sqrt{\frac{3}{\pi}} \frac{\upsilon^{-1/12}}{(\xi - \xi_*^{(1)})^{1/4}\Omega^{1/6}(\alpha + (1-\alpha)\sin^2\xi_*^{(1)})^{1/2}} \begin{pmatrix} \alpha\cos\xi_*^{(1)} \\ \sin\xi_*^{(1)} \end{pmatrix} \\ &\times \left[A_>^{(1)} \exp\left(-\frac{2}{3}\upsilon^{1/2}\Omega(\xi - \xi_*^{(1)})^{3/2} - \frac{5i\pi}{12}\right)\right. \\ &\left. + B_>^{(1)} \exp\left(\frac{2}{3}\upsilon^{1/2}\Omega(\xi - \xi_*^{(1)})^{3/2} + \frac{5i\pi}{12}\right)\right], \end{aligned} \tag{3.5}$$

where $\upsilon = 2\sqrt{-\alpha\varepsilon_\parallel(\varepsilon_a + \alpha\varepsilon_\perp)}$, $A_<^{(1)}$, $B_<^{(1)}$, $A_>^{(1)}$, $B_>^{(1)}$, are constants.

The range of applicability (15) of the WKB solutions intersects with the range of applicability of the solution in the turning point vicinity. The range of intersection is determined by inequalities

$$\Omega^{-2/3} \ll |\xi - \xi_*^{(1)}| \ll 1.$$

In this range the asymptotics of the WKB solution for $|\xi - \xi_*^{(1)}| \ll 1$ should coincide with the asymptotics of the solution near the turning point for $|\xi - \xi_*^{(1)}| \gg \Omega^{-2/3}$.

If we expand the WKB solution (2.7) in series near the point $\xi_*^{(1)}$ then we get

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_<^{(e)\pm} = \frac{C_<^{(e)\pm}}{(v(\xi_*^{(1)}-\xi))^{1/4}(\alpha+(1-\alpha)\sin^2\xi_*^{(1)})^{1/2}} \begin{pmatrix} \alpha\cos\xi_*^{(1)} \\ \sin\xi_*^{(1)} \end{pmatrix} \times \exp\left[\pm i\Omega\left(\int_{\xi_0}^{\xi_*^{(1)}} \frac{k_z^{(e)}(\xi')}{k_0}d\xi' - v^{1/2}\frac{2}{3}(\xi_*^{(1)}-\xi)^{3/2}\right)\right], \quad \xi < \xi_*^{(1)}, \tag{3.6}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_1^{(e)\pm} = \frac{C_1^{(e)\pm}}{(-v(\xi-\xi_*^{(1)}))^{1/4}(\alpha+(1-\alpha)\sin^2\xi_*^{(1)})^{1/2}} \begin{pmatrix} \alpha\cos\xi_*^{(1)} \\ \sin\xi_*^{(1)} \end{pmatrix} \times \exp\left[\pm\Omega\left(i\int_{\xi_0}^{\xi_*^{(1)}} \frac{k_z^{(e)}(\xi')}{k_0}d\xi' - v^{1/2}\frac{2}{3}(\xi-\xi_*^{(1)})^{3/2}\right)\right], \quad \xi > \xi_*^{(1)}. \tag{3.7}$$

Here index "$<$" of the field components and constants refers to the values from the left of the first turning point in the range where WKB approximation is valid. Index "1" refers to the same values from the right of the turning point. If we equate the asymptotics then we get the relations between constants $C^{(e)+}$ and $C^{(e)-}$ corresponding to the left and to the right sides of the turning point. The values of the amplitudes do not need to be equal. The reason is the mutual mode transformations when the wave vectors of the waves approach. Equating the asymptotics of the WKB solutions and the solutions near the turning point we get

$$\begin{pmatrix} C_<^{(e)-} \\ C_1^{(e)+} \end{pmatrix} = \begin{pmatrix} \exp(2s - i\pi/2) & \exp(-i\pi/4) \\ \exp(i\pi/4) & 0 \end{pmatrix} \begin{pmatrix} C_<^{(e)+} \\ C_1^{(e)-} \end{pmatrix}, \tag{3.8}$$

where

$$s = i\Omega\int_{\xi_0}^{\xi_*^{(1)}} \frac{k_z^{(e)}(\xi')}{k_0}d\xi'.$$

Here the constants $C_1^{(e)+}$ and $C_1^{(e)-}$ describe the amplitudes of the WKB solutions after transition of the first turning point.

Moreover there are two turning points inside the forbidden zone. These points, solution (2.16), (points 2 and 3 in Fig. 3) describe the ordinary and extraordinary mode transformation. The vicinities of these turning points are determined by the condition (2.18). In the points ξ_{**} degeneration of $e-o$ type takes place.

Let us consider the interaction of the waves propagating in the positive direction of ξ. It is the interaction of the modes $(e)+$ and $(o)+$ near the point where $\sin\xi_{**}^{(2)} = -\sqrt{-\alpha/(1-\alpha)}$. In this case the solutions of the standard equation are the modified Bessel functions of the first and the second kind, $I_{1/4}(u)$ and $K_{1/4}(u)$.

We complete the matching of the solutions near the turning points with the WKB solutions. Solution of the Eq. (3.3) for $|\Omega^{1/2}(\xi-\xi_{**}^{(2)})|\gg 1$ can be written as

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{>,<}^{(2)} = \frac{\varepsilon_\perp^{1/4}\exp[i\Omega\sqrt{\alpha\varepsilon_\perp}(\xi-\xi_{**}^{(2)})]}{\varepsilon_a^{1/2}\Omega^{1/4}|\xi-\xi_{**}^{(2)}|^{-1/2}} \times\left\{\sqrt{\frac{2}{\pi}}A_{>,<}^{(2)}\exp\left[\frac{1}{2}\Omega\frac{\varepsilon_a}{\sqrt{\varepsilon_\perp}}(\xi-\xi_{**}^{(2)})^2\right]+\sqrt{2\pi}B_{>,<}^{(2)}\right\} \times\begin{pmatrix} \alpha\cos\xi_{**}^{(2)} \\ \sin\xi_{**}^{(2)} \end{pmatrix}. \tag{3.9}$$

Here upper index "(2)" of the field means the vicinity of the turning point $\xi_{**}^{(2)}$. The WKB solution in the vicinity of the point $\xi_{**}^{(2)}$ for the ordinary wave has the form

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{<,2}^{(o)+} = \frac{C_{<,2}^{(o)+}}{(\alpha\varepsilon_\perp)^{1/4}[-2i\alpha^{1/2}(\xi-\xi_{**}^{(2)})]^{1/2}}\begin{pmatrix} \sin\xi_{**}^{(2)} \\ -\cos\xi_{**}^{(2)} \end{pmatrix}\exp[i\Omega\sqrt{\alpha\varepsilon_1}(\xi-\xi_0)], \tag{3.10}$$

where index "<" of the field components and the constants refers to the values from the left of the turning point $\xi_{**}^{(2)}$ in the range where WKB approximation is valid. Index "2" denotes the same values from the right to the turning point $\xi_{**}^{(2)}$. The extraordinary wave in the vicinity of this point is

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{1,2}^{(e)+} = \frac{C_{1,2}^{(e)+}}{(\alpha\varepsilon_\perp)^{1/4}[-2i\alpha^{1/2}(\xi-\xi_{**}^{(2)})]^{1/2}}\begin{pmatrix} \alpha\cos\xi_{**}^{(2)} \\ -\sin\xi_{**}^{(2)} \end{pmatrix} \times\exp\left[i\Omega\int_{\xi_0}^{\xi_{**}^{(2)}}\frac{k_z^{(e)}(\xi)}{k_0}d\xi+i\Omega\sqrt{\alpha\varepsilon_\perp}(\xi-\xi_0)+\frac{1}{2}\Omega\frac{\varepsilon_a}{\sqrt{\epsilon_\perp}}(\xi-\xi_{**}^{(2)})^2\right] \tag{3.11}$$

Here the low index "1" designates the field components and the constants in the region between the turning points $\xi_*^{(1)}$ and $\xi_{**}^{(2)}$ where the WKB approximation is valid. Index "2" shows the values from the right to the turning point $\xi_{**}^{(2)}$ where the WKB approximation is valid.

The amplitudes of the WKB solution in the both sides of the turning point $\xi_{**}^{(2)}$ are connected by the relation

$$\begin{pmatrix} C_2^{(e)+} \\ C_2^{(o)+} \end{pmatrix} = \begin{pmatrix} i & -2^{1/2}\alpha^{-1/2} e^{i\Omega\left[\sqrt{\alpha\varepsilon_\perp}(\xi_{**}^{(2)}-\xi_0)-\int_{\xi_0}^{\xi_{**}^{(2)}}\frac{k_z^{(e)}(\xi)}{k_0}d\xi\right]} \\ 0 & -i \end{pmatrix} \begin{pmatrix} C_1^{(e)+} \\ C_<^{(o)+} \end{pmatrix}. \tag{3.12}$$

The contribution of the ordinary wave into extraordinary one is determined by exponential factor in Eq. (3.12). Its value depends on the difference of the intervals where each of the waves decay. The decay of the extraordinary wave takes place between points $\xi_*^{(1)}$ and $\xi_{**}^{(2)}$ whereas the ordinary wave decays between points ξ_0 and $\xi_{**}^{(2)}$. Therefore the contribution of the ordinary wave into extraordinary one is significant if the difference $\xi_0 - \xi_*^{(1)}$ is small.

The description of the interaction of the waves (e)– and (o)– near the point $\xi_{**}^{(2)}$ and construction the solutions in the vicinities of the turning points $\xi_{**}^{(3)}$ and $\xi_*^{(4)}$ can be completed in a similar way.

The variation of the amplitude of the extraordinary wave is determined by the negative real exponent (2.7) inside the forbidden zone. The transmittance coefficient of the extraordinary wave can be written in the form

$$W = \exp\left(-\Omega\int_{\xi_*^{(1)}}^{\xi_*^{(4)}}\left|\frac{k_z^{(e)}(\xi)}{k_0}\right|d\xi\right). \tag{3.13}$$

In order to apply the developed approach for the wide forbidden zones it is necessary that four turning points were situated far off from each other. In this case the degeneration of the wave vectors takes place in pairs. In this case the equation set (3.3) can be converted to two nonlinked equations of the second order, instead of two linked equation of the second order as it takes place for degeneration of all four wave vectors. Moreover for realization of the sequential procedure of the matching with the WKB solutions it is necessary to have the ranges where the WKB solutions are applicable between the turning points. These conditions impose the restriction on the width of the forbidden zone. According to Eqs. (2.15)–(2.18) the following estimates can be obtained

$$\xi_* \approx \sqrt{-\frac{\alpha}{1-\alpha}\frac{\varepsilon_\|}{\varepsilon_a}} \gtrsim \Omega^{-2/3}, \quad \xi_{**} \approx -\sqrt{-\frac{\alpha}{1-\alpha}} \gtrsim \Omega^{-1/2}, \tag{3.14}$$

$$-\alpha \gtrsim \frac{\varepsilon_a}{\left(\sqrt{\varepsilon_\|} - \sqrt{\varepsilon_a}\right)^2}\Omega^{-1}, \quad -\alpha \gtrsim \Omega^{-1}. \tag{3.15}$$

The behavior of the damping coefficient for the values $\Omega^{-1} < -\alpha \ll 1$ can be found from the relation

$$\begin{aligned} |W|^2 &= \exp\left\{-2\Omega\int_{\xi_*^{(1)}}^{\xi_*^{(4)}} |\alpha\varepsilon_\| + (1-\alpha)\varepsilon_a \sin^2\xi|^{1/2} d\xi\right\} \\ &\approx \exp\left[-2\Omega\varepsilon_a^{1/2}\int_{-\Theta}^{\Theta}(\Theta^2 + \xi^2)d\xi\right], \end{aligned} \tag{3.16}$$

where

$$\Theta = \sqrt{|\alpha|\frac{\varepsilon_\|}{\varepsilon_a}}.$$

Calculating integral we get

$$|W|^2 = \exp\left(-\pi|\alpha|\Omega\frac{\varepsilon_\|}{\varepsilon_a^{1/2}}\right). \tag{3.17}$$

So Eq. (3.17) describes the behavior of the transmission coefficient for the wide forbidden zones, Eq. (3.15).

4. THE NARROW FORBIDDEN ZONES

In this section we consider the case of the beam transition through the narrow forbidden zone, the most important case from the experiment standpoint. The problem in the description of the narrow forbidden zone case is that the turning points approach each other and hence remains no WKB region between them. In this case the standard equation should be constructed for the region which contains all the turning points. We restrict ourselves to the case of the extremely narrow forbidden zones when the turning points can be considered as a single effective point.

In the system with $\varepsilon_a > 0$ the coefficient α is small and negative for narrow forbidden zone and α tends to zero as the width of zone decreases. Further we consider that α is of the order of Ω^{-1}

$$\alpha = -\delta\alpha\Omega^{-1}, \quad 0 < \delta\alpha \lesssim 1, \tag{4.1}$$

where parameter $\delta\alpha$ is connected with the width of the forbidden zone Δ by the relation

$$\delta\alpha \approx \Omega \frac{\varepsilon_a \Delta^2 q_0^2}{4\varepsilon_\parallel}. \tag{4.2}$$

The width of the forbidden zone is related to the angle of incidence of the beam χ_0 (e.g., in the point $\xi = -\pi/2$) by the relation

$$\Delta = \frac{2}{q_0} \arccos\left[\cot\chi_0 \sqrt{\frac{\varepsilon_\perp}{\varepsilon_a}}\right]. \tag{4.3}$$

We consider a narrow forbidden zone Δ when the angle of incidence χ_0 is close to the critical one, $\chi_0^{(c)}$, determined by the relation

$$\cot\chi_0^{(c)} = \frac{\pi}{2}\sqrt{\frac{\varepsilon_a}{\varepsilon_\perp}}.$$

At the condition (4.1) the turning points (2.15) degenerate practically into the point $\xi = 0$. As it follows from Eq. (4.2) in this case the vicinity of the turning point is bounded by inequality

$$|\xi| \lesssim \Omega^{-1/2}. \tag{4.4}$$

For narrow forbidden zones in the case $\varepsilon_a > 0$ difficulties emerge in constructing the successive solution of Eq. (3.3). In this case the problem is that four turning points come close to each other. Formally it leads to the differential equation of the fourth order for each component of the field and the solution of this equation becomes difficult. Therefore we restrict ourselves by the construction of the approximate solution based on the following physical conception: we neglect the interaction of the ordinary and the extraordinary waves. Then the propagation of the extraordinary wave is described by the scalar Helmholtz equation in the medium with periodically varying refractive index. If we take into account the relation (2.9) then in $(\mathbf{k}_{\text{perp}}, z)$ presentation this equation has the form

$$\partial_z^2 \mathcal{E} + k_z^{(e)2} \mathcal{E} = 0$$

or

$$\partial_{\xi}^{2}\mathcal{E}+\Omega^{2}\left[\alpha\varepsilon_{\|}+(1-\alpha)\varepsilon_{a}\sin^{2}\xi\right]\mathcal{E}=0. \tag{4.5}$$

In this case the field components are determined by the relations $E_x = \mathcal{E}\alpha\cos\xi$, $E_y = \mathcal{E}\sin\xi$.

There are two small parameters in Eq. (4.5), ξ and Ω^{-1}. We make the substitution $\zeta = {\varepsilon_a}^{1/4}\Omega^{1/2}\xi$, where the new variable ζ is already not small. If we expand the coefficients of the Eq. (4.5) in the series near the point $\zeta = 0$ then the Eq. (4.5) attains the form

$$\partial_{\zeta}^{2}\mathcal{E}+\left[\zeta^{2}-\delta\alpha\frac{\varepsilon_{\|}}{\varepsilon_{a}^{1/2}}+\Omega^{-1}\left(\delta\alpha\zeta^{2}-\frac{\zeta^{4}}{3\varepsilon_{a}^{1/2}}\right)+\cdots\right]\mathcal{E}=0. \tag{4.6}$$

Its solution is sought as the series over powers of Ω^{-1}

$$\mathcal{E}(\zeta)=\mathcal{E}_{0}(\zeta)+\Omega^{-1}\ \mathcal{E}_{1}(\zeta)+\Omega^{-2}\mathcal{E}_{2}(\zeta)+\cdots. \tag{4.7}$$

Restricting ourself to the higher terms of the equation and zeroth approximation over $\mathcal{E}$ we get

$$\partial_{\zeta}^{2}\mathcal{E}_{0}+[\zeta^{2}-\psi^{2}]\mathcal{E}_{0}=0, \tag{4.8}$$

where

$$\psi=\left[\delta\alpha\frac{\varepsilon_{\|}}{{\varepsilon_{a}}^{1/2}}\right]^{1/2}. \tag{4.9}$$

The solutions of the Eq. (4.8) are the functions of the parabolic cylinder

$$\mathcal{E}_{0}=A_{1}D_{-1/2-i\psi^{2}/2}(\sqrt{2}e^{-i\pi/4}\zeta)+A_{2}D_{-1/2-i\psi^{2}/2}(\sqrt{2}e^{i3\pi/4}\zeta). \tag{4.10}$$

In order to compare the obtained result with the WKB solution we use the asymptotic of the functions of the parabolic cylinder for large argument $|u/\nu|\gg 1$

$$D_{\nu}(u)=\begin{cases} e^{-u^{(2)/4}}u^{\nu}[1+O(u^{-2})], \quad \arg u\in(-\pi/2,\pi/2) \\ \left[e^{-u^{(2)/4}}u^{\nu}-\sqrt{2\pi}\Gamma^{-1}(-\nu)e^{u^{(2)/4}\pm i\pi\nu}u^{-1-\nu}\right][1+O(u^{-2})], \\ \text{for sign}''+''\arg u\in(\pi/2,\pi);\quad \text{for sign}''-''\arg u\in(-\pi,-\pi/2) \end{cases} \tag{4.11}$$

For the wave incident from the side $\zeta\longrightarrow-\infty$, there exist two waves in the region $\zeta<0$, the incident and the reflected one, (Fig. 2(b)). Behind the barrier, $\zeta \longrightarrow +\infty$, there exists the outgoing wave only. As far as in Eq. (4.10) the term proportional to A_2 produces two waves in the region $\zeta > 0$ it is necessary to assume that $A_2 = 0$.

Thus asymptotic of the solution (4.10) has the form

$$\mathcal{E}_{0,>} = G_3 \zeta^{-1/2} \exp\left[i \frac{\psi^2}{2} \left(\frac{\zeta^2}{\psi^2} - \ln \frac{2\zeta}{|\psi|} \right) \right] \tag{4.12}$$

for $\zeta > 0$, and

$$\mathcal{E}_{0,<} = |\zeta|^{-1/2} \left\{ G_2 \exp\left[i \frac{\psi^2}{2} \left(\frac{\zeta^2}{\psi^2} - \ln \frac{2|\zeta|}{|\psi|} \right) \right] \right. \\ \left. + G_1 \exp\left[-i \frac{\psi^2}{2} \left(\frac{\zeta^2}{\psi^2} - \ln \frac{2|\zeta|}{|\psi|} \right) \right] \right\} \tag{4.13}$$

for $\zeta < 0$, where

$$G_1 = \sqrt{2\pi}\Gamma^{-1}\left(\frac{1}{2} + i \frac{\psi^2}{2} \right) \exp\left[i \left(\frac{\pi}{8} - \frac{\psi^2}{4} \ln 2 + \frac{\psi^2}{2} \ln|\psi| \right) - \frac{1}{4} \ln 2 + \frac{\pi\psi^2}{8} \right]$$

$$G_2 = \exp\left[-i \left(\frac{3\pi}{8} - \frac{\psi^2}{4} \ln 2 + \frac{\psi^2}{2} \ln|\psi| \right) - \frac{1}{4} \ln 2 + \frac{3\pi\psi^2}{8} \right]$$

$$G_3 = \exp\left[i \left(\frac{\pi}{8} + \frac{\psi^2}{4} \ln 2 - \frac{\psi^2}{2} \ln|\psi| \right) - \frac{1}{4} \ln 2 - \frac{\pi\psi^2}{8} \right].$$

It is possible to ascertain that this asymptotic corresponds to the higher term of the WKB approximation as far as

$$\int_{\psi}^{\zeta} (\zeta^2 - \psi^2)^{1/2} d\zeta = \frac{\psi^2}{2} \left[\frac{\zeta}{\psi} \sqrt{\frac{\zeta^2}{\psi^2} - 1} - \ln\left(\frac{\zeta}{\psi} + \sqrt{\frac{\zeta^2}{\psi^2} - 1} \right) \right]. \tag{4.14}$$

We calculate the coefficients of reflection $V = G_2/G_1$ and transmission $W = G_3/G_1$

$$\begin{aligned} V &= \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2} + i \frac{\psi^2}{2} \right) \exp\left[-i \frac{\pi}{2} + i \frac{\psi^2}{2} \ln 2 - i\psi^2 \ln|\psi| + \frac{\pi\psi^2}{4} \right] \\ W &= \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2} + i \frac{\psi^2}{2} \right) \exp\left[+i \frac{\psi^2}{2} \ln 2 - i\psi^2 \ln|\psi| - \frac{\pi\psi^2}{4} \right]. \end{aligned} \tag{4.15}$$

Using the relation for Gamma-functions

$$|\Gamma(1-b+i\alpha)\Gamma(b+i\alpha)| = \pi|\sin\pi(b+i\alpha)|^{-1}. \tag{4.16}$$

we get

$$V = \left[1+\exp(-\pi\psi^2)\right]^{-1/2}\exp[-i(\pi/2+\phi)]$$
$$W = \left[1+\exp(\pi\psi^2)\right]^{-1/2}\exp[-i\phi], \tag{4.17}$$

where

$$\phi = -\arg\Gamma(1/2+i\psi^2/2) - \psi^2\ln 2/2 + \psi^2\ln|\psi|.$$

For energies of reflected and transmitted waves we get

$$|V|^2 = (1+e^{-\pi\psi^2})^{-1}, \quad |W|^2 = (1+e^{\pi\psi^2})^{-1}. \tag{4.18}$$

It is easy to check that the sum of the reflection and of the transmission coefficients (4.18) is equal to unity.

Note that obtained solutions are valid when the component of the wave vector $k_z^{(e)}$ is not reduced to zero but its value is very small. In this case the WKB approximation is also violated and it is necessary to use the standard equation. This case differs from previous one by the sign of inequality, $\delta\alpha < 0$, and the coefficient ψ is imaginary. It corresponds to the reflection above the barrier in quantum mechanics (Fig. 4).

The angular dependence of the reflection and the transmission coefficients for various ratios of the pitch to wave length is showed in Figure 5. There is no forbidden zone for the angle of incidence $\chi_2 < \chi_2^{(0)} \approx 63.45^\circ$ (for the set of parameters in Fig. 5) and extraordi-

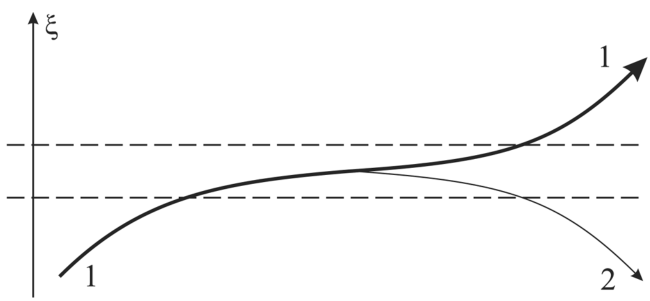

FIGURE 4 The trajectory of the extraordinary beam close to the critical one when the forbidden zone does not exist yet (1). The reflected beam (2) caused by the reflection above the barrier. The dotted lines show the region where the WKB approximation is not valid.

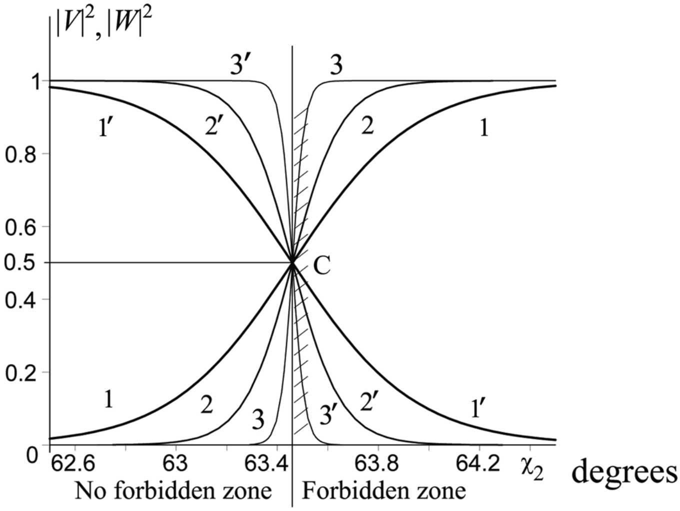

FIGURE 5 Angular dependance of the reflection $|V|^2$ (lines 1, 2, 3) and transmission $|W|^2$ (lines 1′, 2′, 3′) coefficients in the chiral media with $\varepsilon_{\parallel} = 2.86$, $\varepsilon_{\perp} = 2.29$ for different values of $\Omega = p_0/\lambda$: $1 - \Omega = 20$, $2 - \Omega = 50$, $3 - \Omega = 200$, χ_2 is the angle between the beam and the z axis on the plane $\xi = -\pi/2$. The critical beam corresponds to $\chi_2 = \arcsin\sqrt{\varepsilon_{\perp}/\varepsilon_{\parallel}} = 63.46°$. The shaded area shows the presence of the forbidden zone. All the curves intersect in the point C, corresponding to the critical beams.

nary beam passes through the medium with the reflection above the barrier for angles of incidence close to $\chi_2^{(0)}$. The angle $\chi_2^{(0)}$ corresponds to the zeroth forbidden zone, where intensities of the reflected and the transmitted beams are equal regardless of Ω. For angles of incidence $\chi_2 > \chi_2^{(0)}$ the transmission through the forbidden zone takes place and the intensity of the reflected beam is greater than the intensity of the transmitted beam. The moderate values of Ω are more convenient for experiments as far as the effects of reflection above the barrier and transmission through the barrier can be observed in wider range of angles.

5. DISCUSSION

We consider the light propagation in chiral uniaxial media with the pitch considerably exceeding the wave length. For large angles of incidence it exists in these systems the turn back of the extraordinary beam which means the existence of the forbidden zones. We studied the transmission of the beam through the forbidden zone. In this range it is necessary to consider the light field as a whole instead of considering the ordinary and the extraordinary waves of the geometric optics. In practice this description is realized by introducing the standard equation. As far as the form of the wave surface of the extraordinary beam depends on the sign of the permittivity anisotropy ε_a the transmission through the forbidden zone is described differently for systems with positive and negative ε_a. In general the case $\varepsilon_a > 0$ is more complicated since the interaction of the ordinary and the extraordinary beams is possible inside the forbidden zone. For narrow forbidden zones when the turning points merge the transmission problem is being solved approximately. It is interesting to note that this problem is similar to the quantum problem of the electron transmission through the parabolic barrier. Within the framework of this approximation we calculate the reflection and the transmission coefficients for reflection above the barrier and transmission through the forbidden zone.

The effect of transmission through the forbidden zone was studied experimentally in nematic liquid crystal medium doped by the chiral addition. The dependence of the intensity of the transmitted extraordinary beam on angle of incidence is in a good agreement with the results of the theoretical calculations. Due to special form of the liquid crystal cell we succeeded to check the theoretical prediction concerning for equality of the reflected and transmitted beams intensities in the case of extremely narrow forbidden zone.

Of considerable interest is also the following study of the effect of transmission through the forbidden zone in a chiral liquid crystal. Simultaneous measurements of the intensities of the transmitted and the reflected beams were yet not performed. It would be also important to study the transmission of the beam through the wide forbidden zones. In particular these measurements are interesting for theoretical analysis since it is necessary to take into account several turning points simultaneously. The form and the solution of the standard equation in this case case need further analysis.

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